Mathematical Model Written by The Canonical system for some Electrical Rectifier circuits using Semiconductor diodes

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Abstract

The paper investigates Volt-Ampere characteristics of rectifiers us- ing semiconductor diodes, which are popular in electric domain, for example using in half-wave rectification and full-wave rectification. We argue that these characteristics can be presented by a mathematical model using a canonical system. This finding will enable a mathematical, systematic overview of functioning principles of such rectifiers. Based on this, we propose an optimal process for an assembly line of rectifiers in electrical engineering.

Key words: mathematical model, canonical system, Volt–Ampere characteristic, rectifier, half–wave, full–wave.

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1 Introduction

The emergence of mathematical models has addressed a large number of applied problems, such as mechanics, electricity, theory of automation and control, struggle for survival in ecological systems, ... Mathematics is the tool for describing changes in each domain as dynamic systems, through which one can indicate their characteristics. This area is cared by many famous mathematicians in the world such as MA Krasnoselskii, AV Pokroskii, BN Sadovskii, TS Gilman, J. Appell, ID Mayergoyz, MP Sobolev, EA Lifshitz, ... (see [1] - [4]). Currently, the research in this area is still very strongly developed. One of the problems that attracts attention is to study by mathematical modelling the operation of the rectifier circuit [5] - [11].

As we already know, most electronic installations use direct current (DC), but the power source is alternating current (AC). Therefore, the rectifying circuit is very important, indispensable and widely used in the electrical industry.

A rectifier circuit is an electrical circuit consisting of electrical and electronic components used to convert AC to DC [12] - [15]. The positive elements in the rectifier circuit are semiconductor diodes. A semiconductor diode or diode is a type of semiconductor device that allows the current to flow through it in a single direction: from the anode to the cathode and without reversedirection. The diodes are designated as follows (Figure 1):

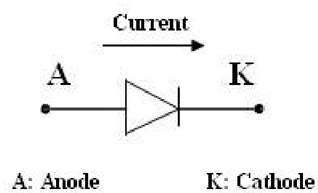

Figure 1. Scheme of the semiconductor diode

Volt-Ampere characteristic of diodes is a graph describing the relationship between current through diodes according to the voltage applied to it. The following describes the Volt–ampere characteristics of an ideal semiconductor diode (Figure 2):

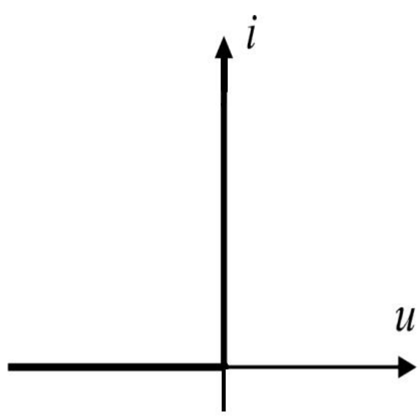

Figure 2. The Volt–ampere characteristics of the semiconductor diode

Suppose that x and y alternately are the intensity of the current from the anode to the cathode set on the diodes. Then the characteristic of semiconductor diodes can be represented as follows:

$$i \geq 0,\ u \leq 0,\ iu = 0. \tag{1}$$

Depending on the voltage, current and properties of electricity need to rectify, the different rectifiers circuits are used. For example, as for big current, we need rectify the capacity, these sets of rectifiers mainly rectify the power supply circuits for electrical and electronic devices.

For the media sources using a transformer, they choose a commonly-used rectifier diode. But for the high frequency pulse sources, the high-speed switching diodes must be used. And the very high voltage sources need the voltage diode. For very small currents, there are the rectifier circuits detector in the AM radio.

In brief, the most basic application of the rectifier circuit is to extract the useful DC component from the alternating current source because in fact, most electronic devices use DC source but the power supply source are alternating current.

In this paper, we will study the appropriate mathematical model for some common rectifier circuits in the science of electrical engineering, these are half–wave rectifying circuit and full–wave rectifying circuit.The mathematical model will be written by the canonical system which has the following form:

$$\begin{cases} \dfrac{dX}{dt} + \mathcal{M}X + Y = \mathcal{F}(t), \\ X \in E, \\ Y \in E^*, \\ (X, Y) = 0 \end{cases} \tag{2}$$

Here: sets E and E^* are conjugate cones in the space $\mathbb{R}^n$; $X(t)$ and $Y(t)$ are unknown functions whose values belong to $\mathbb{R}^n$ at moment t; $\mathcal{M}$ is a known constant square matrix of order n; $\mathcal{F}(t)$ is a known continuous vector-function with its values in $\mathbb{R}^n$.

The solution $X(t)$ of the system (2) is understood as a locally absolutely function which satisfies (2) almost everywhere.

The mathematical model written by (2) will accurately reflect the nature of the electrical circuit and bring many benefits in studying the operation of the rectifier circuit.

2 The mathematical model for the halfwave rectifiers

If in the rectifier circuit, there is only one semiconductor diode, it is the half-wave rectifier circuit. In this type of circuit, there is only half a cycle (positive or negative) that can pass the diode, however, the rest half cycle will be blocked.

It depends on the direction of installation of the diode. And as, only half a cycle is rectified, the half–wave rectifier circuit achieves a very low efficiency of power transmission. The half–wave rectifier circuit includes a diode that is connected to a source with voltage $E(t)$, resistance R and bobbin L (shown in the figure 3).

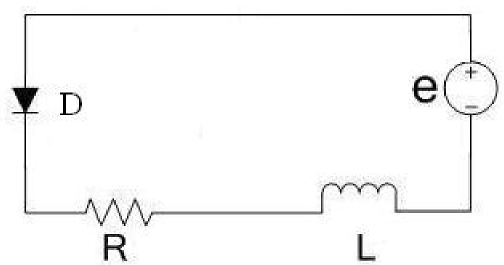

Figure 3. The half–wave rectifier

Suppose that I and U are in turn the intensity of the current and the voltage which passes from the positive pole to the negative pole of the diode. Then

$$L\frac{dI}{dt} + RI + U = E(t). \tag{3}$$

If we denote $X = I$, $\mathcal{A} = \frac{R}{L}$, $\mathcal{E}(t) = \frac{E(t)}{L}$ and $Y = \frac{U}{L}$ then the equation (3) can be written as

$$\dot{X} + \mathcal{A}X + Y = \mathcal{E}(t). \tag{4}$$

On the other hand, according to the principle of the operation of the diodes (see [1]), we see that the vector Y defined on the closed convex set $Q^* = (-\infty, 0] \subset \mathbb{R}$, can be written explicitly as following form:

$$\begin{cases} Y \in \{0\}, \text{ if } X > 0 \\ Y \in (-\infty, 0], \text{ if } X = 0. \end{cases}$$

Summary, the operation of the half-wave rectifier circuit will be written in the form of the canonical system of (2), that mean

$$\begin{cases} \dot{X} + \mathcal{A}X + Y = \mathcal{E}(t), \\ x \in Q \\ y \in Q^* \\ (X, Y) = 0. \end{cases} \tag{5}$$

where $Q = [0, +\infty)$.

3 The mathematical model for the full wave rectifiers

The full–wave rectifier circuit modifies all the polarity components from the initial waveform to a single direction. Therefore, the full–wave rectifier circuit achieves very high efficiency and is used a lot in the technique and in everyday life. The full-wave rectifier circuit comprises four semiconductor diodes; a source with the voltage $e(t)$, the resistance r and the inductance l; the power consumption circuit with the resistor R and the inductance L. The circuit parameters are clearly shown in the following figure:

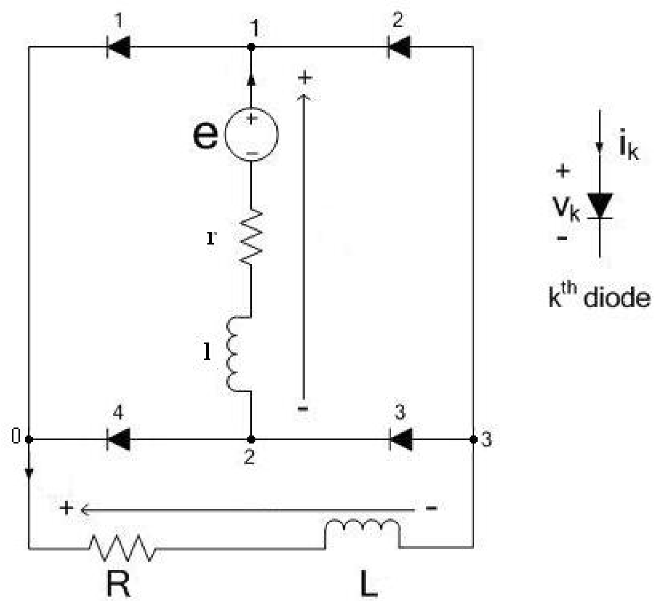

Figure 4. The full–wave rectifier

For this rectifier circuit, we note the current and the voltage (from positive to negative) of the diodes $j(j = \overline{1,4})$ are x_j and y_j respectively. Then, by the operating principle of the diodes, we have:

$$x_j \geq 0,\ y_j \leq 0,\ (x_j, y_j) = 0,\ j = \overline{1,4}. \tag{6}$$

In addition, all the nodes through which the diodes can be connected to other circuits are numbered 0, 1, 2, 3 as shown in the figure. The intensity of the current passing the $k-$node $k = \overline{0,3}$ are denoted by i_k. The voltage between node k and node 0 are denoted by u_k.

In following, we show the operation of the full wave rectifier circuit for the nonlinear analytical model of (2). Assuming that the direction of the current is marked as in the figure, then according to Kirchhoff's law, the operation of the circuit is represented by the following equations:

$$\begin{cases} l\dfrac{di_1}{dt} + ri_1 + u_1 - u_2 = e(t) \\ L\dfrac{di_3}{dt} + Ri_3 + u_3 = 0 \\ i_2 = i_3 - i_1. \end{cases} \tag{7}$$

If we define $I_L = \begin{pmatrix} i_1 \\ i_3 \end{pmatrix}$ then the first equation of the system (7) will be rewritten as following equation:

$$\mathcal{L}\frac{dI_L}{dt} + \mathcal{B}I_L + \mathcal{H}u_D = E(t), \tag{8}$$

where

$$u_D = (u_1, u_2, u_3), \mathcal{L} = \begin{pmatrix} l & 0 \\ 0 & L \end{pmatrix}, \mathcal{B} = \begin{pmatrix} r & 0 \\ 0 & R \end{pmatrix},$$

$$\mathcal{H} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } E(t) = \begin{pmatrix} e(t) \\ 0 \end{pmatrix}$$

We use this following transform:

$$X = \mathcal{L}^{\frac{1}{2}} I_L \text{ and } Y = \mathcal{L}^{-\frac{1}{2}}(\mathcal{H}u_D). \tag{9}$$

Then, the operation of the full–wave rectifier circuit will be rewritten by the following differential equation:

$$\frac{dX}{dt} + \mathcal{J}X + Y = \mathcal{E}(t), \tag{10}$$

where

$$\mathcal{J} = \begin{pmatrix} \frac{r}{l} & 0 \\ 0 & \frac{R}{L} \end{pmatrix}, \mathcal{E}(t) = \begin{pmatrix} \frac{e(t)}{\sqrt{l}} \\ 0 \end{pmatrix} \tag{11}$$

The operation of the full–wave rectifier circuit written by (10) will have the form of the canonical system of (2), if we prove the existence of a conic K in the $\mathbb{R}^2$ space such that

$$\begin{cases} X \in K \\ Y \in K^* \\ (X, Y) = 0. \end{cases} \tag{12}$$

Indeed, according to Kirchhoff's first law, we have

$$\begin{cases} i_1 = x_1 - x_2 = x_3 - x_4 \\ i_3 = x_1 + x_4 = x_2 + x_3. \end{cases} \tag{13}$$

From system (13), it is easy to see that:

$$\begin{cases} i_3 - i_1 = x_4 + x_2 \geq 0 \\ i_3 + i_1 = x_1 + x_3 \geq 0. \end{cases}$$

Thus,

$$i_3 \geq |i_1| > 0. \tag{14}$$

On other hand, by Kirchhoff's second law, we have

$$\begin{cases} u_1 - u_2 = y_1 - y_4 = y_3 - y_2 \\ u_3 = y_1 + y_2 = y_3 + y_4. \end{cases} \tag{15}$$

From the system (15) we have:

$$\begin{cases} u_3 + u_1 - u_2 = y_1 + y_3 \leq 0 \\ u_3 - (u_1 - u_2) = y_2 + y_4 \leq 0. \end{cases}$$

Therefore

$$u_3 \leq -|u_1 - u_2| < 0. \tag{16}$$

From (9), (13) and (15), we have:

$$\begin{aligned} (X, Y) &= i_1(u_1 - u_2) + i_3 u_3 \\ &= \frac{1}{4}\Big[(x_1 - x_2 + x_3 - x_4)(y_1 - y_4 + y_3 - y_2) + (x_1 + x_4 + x_2 + x_3)(y_1 + y_2 + y_3 + y_4)\Big] \end{aligned}$$

It follows:

$$(X, Y) = \frac{1}{2}(x_2 y_4 + x_1 y_3 + x_3 y_1 + x_4 y_2). \tag{17}$$

By virtue of (6) we can obtain that $(X, Y) \leq 0$. To justify

$$(X, Y) = 0, \tag{18}$$

we have to prove that each term on the right-hand side of (17) is equal to 0. We suppose an absurd, then:

$$x_2 y_4 < 0 \Leftrightarrow \begin{cases} x_2 > 0 \Rightarrow y_2 = 0 \\ y_4 < 0 \Rightarrow x_4 = 0 \Rightarrow x_1 = i_3 \Rightarrow y_1 = 0 \end{cases}$$

Hence, in conjunction with (15), it is easy to deduce $u_3 = y_1 + y_2 = 0$. This contradicts with (16), that mean, $x_2 y_4 = 0$. The remaining terms of the right-hand side of (17)) are equal to 0.
Now, we define the cone $K \subset \mathbb{R}^2$ such that $X \in K$. From the transformation (9) and the estimate (14), it is easy to deduce:

$$X \in K = \left\{ \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \in \mathbb{R}^2 : X_2 > 0, X_2 \geq \sqrt{\frac{L}{l}} X_1, X_2 \geq -\sqrt{\frac{L}{l}} X_1 \right\}. \tag{19}$$

Under the modification of variables (9) and the evaluation (16), we have:

$$Y \in K^* = \left\{ \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \in \mathbb{R}^2 : Y_2 < 0, Y_2 \leq \sqrt{\frac{l}{L}} Y_1, Y_2 \leq -\sqrt{\frac{l}{L}} Y_1 \right\}. \tag{20}$$

From (10), (18), (19) and (20), we can confirm that the operation of the fullwave rectifier circuit can be written by the canonical system.
as following:

$$\begin{cases} \dfrac{dX}{dt} + \mathcal{J}X + Y = \mathcal{E}(t) \\ X \in K \\ Y \in K^* \\ (X, Y) = 0, \end{cases} \tag{21}$$

where $\mathcal{J}$ and $\mathcal{E}(t)$ defined by (11).

Thus, by the system (21) can be studied the operation of the full–wave rectifier circuit, absolutely, from a mathematical point of view.

4 Conclusions

This subject study the mathematical model for two rectifier circuits: half–wave rectifier circuit and full–wave rectifier circuit. These are two rectifier circuits common in the science of electrical engineering. The mathematical model is written by a special system of differential equations which is called the canonical system. The built model accurately reflects the nature of the electrical circuit so this can bring benefits in studying these rectifier circuits.

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